RATIOS, PROPORTIONS AND PERCENTS
7 TH GRADE

Lesson 1: An Experience in Relationships as Measuring Rate

Key Terms from Grade 6 Ratios and Unit Rates

A **ratio** is an ordered pair of non-negative numbers, which are not both zero. The ratio is denoted $A: B$ to indicate the order of the numbers: the number A is first and the number B is second.

Two ratios $A: B$ and $C: D$ are **equivalent ratios** if there is a positive number, c, such that $C = cA$ and $D = cB$.

A ratio of two quantities, such as 5 miles per 2 hours, can be written as another quantity called a **rate**.

A **unit rate** is a rate per 1 item/unit such as 2.5 miles/hour, read miles per hour. This means that in 1 hour the car travels 2.5 miles.

Example 1: Our Class by Gender

	Number of boys	Number of girls	Ratio of boys to girls
Class 1			
Class 2			
Whole 7^{th} Grade			

Create a pair of equivalent ratios by making a comparison of quantities discussed in this Example.

Exercise 1: Which is the Better Buy?

Value-Mart is advertising a Back-to-School sale on pencils. A pack of 30 sells for \$7.97, whereas a 12-pack of the same brand cost for \$4.77. Which is the better buy? How do you know?

Lesson 2: Proportional Relationships

Classwork

Example 1: Pay by the Ounce Frozen Yogurt!

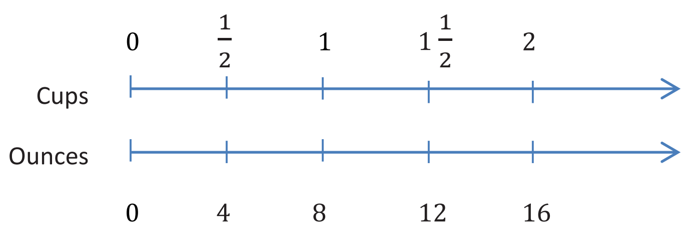
A new self-serve frozen yogurt store opened this summer that sells its yogurt at a price based upon the total weight of the yogurt and its toppings in a dish. Each member of Isabelle's family weighed their dish and this is what they found. Determine if the cost is proportional to the weight.

Weight (ounces)	12.5	10	5	8
Cost ($)	5	4	2	3.20

The cost ____ the weight.

Example 2: A Cooking Cheat Sheet!

In the back of a recipe book, a diagram provides easy conversions to use while cooking.

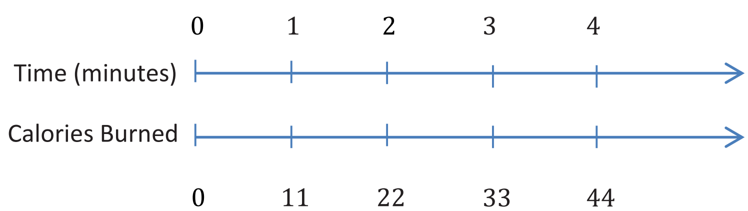


The ounces ____the cups.

Exercise 1

During Jose's physical education class today, students visited activity stations. Next to each station was a chart depicting how many calories (on average) would be burned by completing the activity.

Calories Burned while Jumping Rope



a. Is the number of calories burned proportional to time? How do you know?

b. If Jose jumped rope for 6.5 minutes, how many calories would he expect to burn?

Example 3: Summer Job

Alex spent the summer helping out at his family's business. He was hoping to earn enough money to buy a new $220 gaming system by the end of the summer. Halfway through the summer, after working for 4 weeks, he had earned $112. Alex wonders, "If I continue to work and earn money at this rate, will I have enough money to buy the gaming system by the end of the summer?"

To determine if he will earn enough money, he decided to make a table. He entered his total money earned at the end of Week 1 and his total money earned at the end of Week 4.

Week	0	1	2	3	4	5	6	7	8
Total Earnings		$28			$112				

a. Work with a partner to answer Alex's question.

b. Are Alex's total earnings proportional to the number of weeks he worked? How do you know?

Lesson 3: Identifying Proportional and Non-Proportional Relationships in Tables

Classwork

Example

You have been hired by your neighbors to babysit their children on Friday night. You are paid $8 per hour. Complete the table relating your pay to the number of hours you worked.

Hours Worked	Pay
1	
2	
3	
4	
$4\frac{1}{2}$	
5	
6	
6.5	

Based on the table above, is the pay proportional to the hours worked? How do you know?

Exercises

For Exercises 1–3, determine if y is proportional to x. Justify your answer.

1. The table below represents the relationship of the amount of snowfall (in inches) in 5 counties to the amount of time (in hours) hours of a recent winter storm.

x Time (h)	y Snowfall (in.)
2	10
6	12
8	16
2.5	5
7	14

2. The table below shows the relationship between the cost of renting a movie (in dollars) to the number of days the movie is rented.

x Number of Days	y Cost (dollars)
6	2
9	3
24	8
3	1

3. The table below shows the relationship between the amount of candy bought (in pounds) and the total cost of the candy (in dollars).

x Amount of Candy (pounds)	y Cost (dollars)
5	10
4	8
6	12
8	16
10	20

4. Randy is planning to drive from New Jersey to Florida. Every time Randy stops for gas, he records the distance he traveled in miles and the total number of gallons used.

   Assume that the number of miles driven is proportional to the number of gallons consumed in order to complete the table.

Gallons Consumed	2	4		8	10	12
Miles Driven	54		189	216		

Lesson 4: Identifying Proportional and Non-Proportional Relationships in Tables

Classwork

Example: Which Team Will Win the Race?

You have decided to walk in a long distance race. There are two teams that you can join. Team A walks at a constant rate of 2.5 miles per hour. Team B walks 4 miles the first hour and then 2 miles per hour after that.

Task: Create a table for each team showing the distances that would be walked for times of 1, 2, 3, 4, 5, and 6 hours. Using your tables, answer the questions that follow.

Team A	
Time (h)	Distance (miles)

Team B	
Time (h)	Distance (miles)

a. For which team is distance proportional to time? Explain your reasoning.

b. Explain how you know distance for the other team is not proportional to time.

c. At what distance in the race would it be better to be on Team B than Team A? Explain.

d. If the members on each team ran for 10 hours, how far would each member run on each team?

e. Will there always be a winning team, no matter what the length of the course? Why or why not?

f. If the race is 12 miles long, which team should you choose to be on if you wish to win? Why would you choose this team?

g. How much sooner would you finish on that team compared to the other team?

Exercises

5. Bella types at a constant rate of 42 words per minute. Is the number of words she can type proportional to the number of minutes she types? Create a table to determine the relationship.

Minutes	1	2	3	6	60
Number of Words					

6. Mark recently moved to a new state. During the first month he visited five state parks. Each month after he visited two more. Complete the table below and use the results to determine if the number of parks visited is proportional to the number of months.

Number of Months	Number of State Parks
1	
2	
3	
	23

7. The table below shows the relationship between the side length of a square and the area. Complete the table. Then determine if the length of the sides is proportional to the area.

Side Length (inches)	Area (square inches)
1	1
2	4
3	
4	
5	
8	
12	

Lesson 5: Identifying Proportional and Non-Proportional Relationships in Graphs

Classwork

Opening Exercise

Isaiah sold candy bars to help raise money for his scouting troop. The table shows the amount of candy he sold compared to the money he received.

x Candy Bars Sold	y Money Received ($)
2	3
4	5
8	9
12	12

Is the amount of candy bars sold proportional to the money Isaiah received? How do you know?

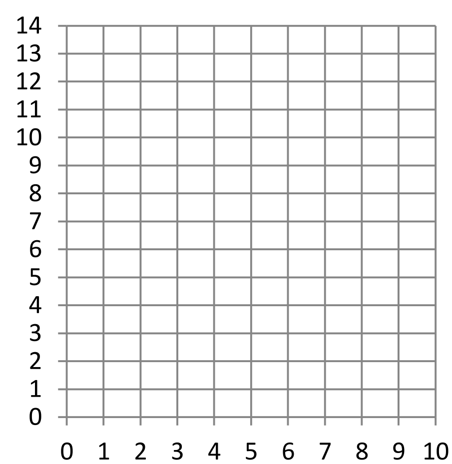
____

____

Example 1: From a Table to Graph

Using the ratio provided, create a table that shows money received is proportional to the number of candy bars sold. Plot the points in your table on the grid.

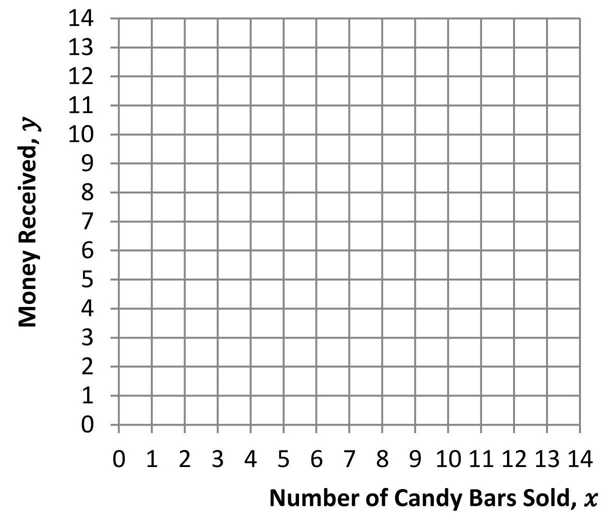
x Candy Bars Sold	y Money Received ($)
2	3



Example 2

Graph the points from the Opening Exercise.

x Candy Bars Sold	y Money Received ($)
2	3
4	6
8	12
12	14



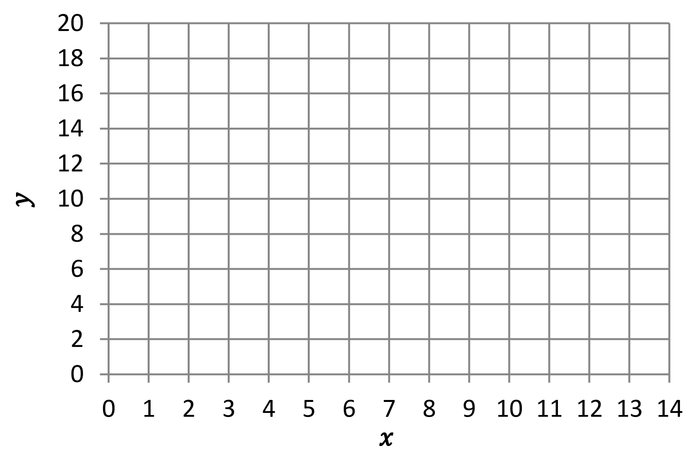
Important Note:

Characteristics of graphs of proportional relationships:

Example 3

Graph the points provided in the table below and describe the similarities and differences when comparing your graph to the graph in Example 1.

x	y
0	6
3	9
6	12
9	15
12	18



Similarities with Example 1:

Differences from Example 1:

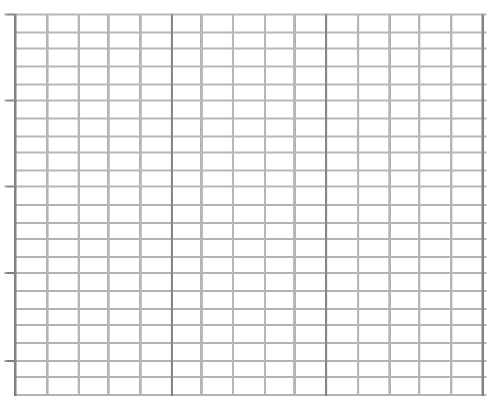
Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Classwork

Today's Exploratory Challenge is an extension of Lesson 5. You will be working in groups to create a table and graph and identify whether the two quantities are proportional to each other.

Poster Layout

Use for notes

Problem:	Table:
Graph: 	Proportional or Not? Explanation:

Gallery Walk

Take notes and answer the following questions:

- Were there any differences found in groups that had the same ratios?
- Did you notice any common mistakes? How might they be fixed?
- Were there any groups that stood out by representing their problem and findings exceptionally clearly?

Poster 1:

Poster 2:

Poster 3:

Poster 4:

Poster 5:

Poster 6:

Poster 7:

Poster 8:

Note about Lesson Summary:

Lesson 7: Unit Rate as the Constant of Proportionality

Classwork

Example 1: National Forest Deer Population in Danger?

Wildlife conservationists are concerned that the deer population might not be constant across the National Forest. The scientists found that there were 144 deer in a 16 square mile area of the forest. In another part of the forest, conservationists counted 117 deer in a 13 square mile area. Yet a third conservationist counted 216 deer in a 24 square mile plot of the forest. Do conservationists need to be worried?

a. Why does it matter if the deer population is not constant in a certain area of the National Forest?

b. What is the population density of deer per square mile?

The unit rate of deer per 1 square mile is _______.

Constant of Proportionality:

Explain the meaning of the constant of proportionality in this problem:

c. Use the unit rate of deer per square mile (or $\frac{y}{x}$) to determine how many deer are there for every 207 square miles.

d. Use the unit rate to determine the number of square miles in which you would find 486 deer?

Vocabulary:

A **constant** specifies a unique number.

A **variable** is a letter that represents a number.

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is a positive constant, then k is called the **constant of proportionality**. It is the value that describes the multiplicative relationship between two quantities, x and y. The (x, y) pairs represent all the pairs of values that make the equation true.

Note: In a given situation, it would be reasonable to assign any variable as a placeholder for the given quantities. For example, a set of ordered pairs (t, d) would be all the points that satisfy the equation $d = rt$, where r is the positive constant, or the constant of proportionality. This value for r specifies a unique number for the given situation.

Example 2: You Need WHAT???

Brandon came home from school and informed his mother that he had volunteered to make cookies for his entire grade level. He needs 3 cookies for each of the 96 students in 7th grade. Unfortunately, he needs the cookies the very next day! Brandon and his mother determined that they can fit 36 cookies on two cookie sheets.

a. Is the number of cookies proportional to the number of cookie sheets used in baking? Create a table that shows data for the number of sheets needed for the total number of cookies baked.

Table:

The unit rate of $\frac{y}{x}$ is __________.

Constant of Proportionality:

Explain the meaning of the constant of proportionality in this problem:

b. It takes 2 hours to bake 8 sheets of cookies. If Brandon and his mother begin baking at $4{:}00$ p.m., when will they finish baking the cookies?

Example 3: French Class Cooking

Suzette and Margo want to prepare crêpes for all of the students in their French class. A recipe makes 20 crêpes with a certain amount of flour, milk, and 2 eggs. The girls already know that they have plenty of flour and milk to make 50 crêpes, but they need to determine the number of eggs they will need for the recipe because they are not sure they have enough.

a. Considering the amount of eggs necessary to make the crêpes, what is the constant of proportionality?

b. What does the constant or proportionality mean in the context of this problem?

c. How many eggs are needed to make 50 crêpes?

Lesson 8: Representing Proportional Relationships with Equations

Classwork

Points to remember:

- Proportional relationships have a constant ratio, or unit rate.
- The constant ratio, or unit rate of $\frac{y}{x}$, can also be called the constant of proportionality.

Discussion Notes

How could we use what we know about the constant of proportionality to write an equation?

Example 1: Do We have Enough Gas to Make it to the Gas Station?

Your mother has accelerated onto the interstate beginning a long road trip and you notice that the low fuel light is on, indicating that there is a half a gallon left in the gas tank. The nearest gas station is 26 miles away. Your mother keeps a log where she records the mileage and the number of gallons purchased each time she fills up the tank. Use the information in the table below to determine whether you will make it to the gas station before the gas runs out. You know that if you can determine the amount of gas that her car consumes in a particular number of miles, then you can determine whether or not you can make it to the next gas station.

Mother's Gas Record

Gallons	Miles driven
8	224
10	280
4	112

a. Find the constant of proportionality and explain what it represents in this situation.

b. Write equation(s) that will relate the miles driven to the number of gallons of gas.

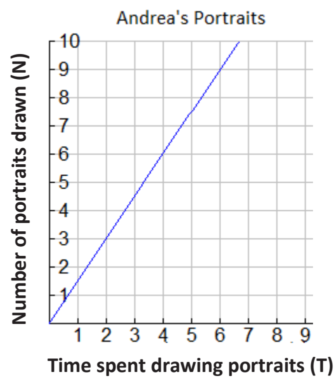
c. Knowing that there is a half-gallon left in the gas tank when the light comes on, will she make it to the nearest gas station? Explain why or why not.

d. Using the equation found in part (b), determine how far your mother can travel on 18 gallons of gas. Solve the problem in two ways: once using the constant of proportionality and once using an equation.

e. Using the constant of proportionality, and then using the equation found in part (b), determine how many gallons of gas would be needed to travel 750 miles.

Example 2: Andrea's Portraits

Andrea is a street artist in New Orleans. She draws caricatures (cartoon-like portraits) of tourists. People have their portrait drawn and then come back later to pick it up from her. The graph below shows the relationship between the number of portraits she draws and the amount of time in hours she needs to draw the portraits.



a. Write several ordered pairs from the graph and explain what each ordered pair means in the context of this graph.

b. Write several equations that would relate the number of portraits drawn to the time spent drawing the portraits.

c. Determine the constant of proportionality and explain what it means in this situation.

Lesson 9: Representing Proportional Relationships with Equations

Classwork

Example 1: Jackson's Birdhouses

Jackson and his grandfather constructed a model for a birdhouse. Many of their neighbors offered to buy the birdhouses. Jackson decided that building birdhouses could help him earn money for his summer camp, but he is not sure how long it will take him to finish all of the requests for birdhouses. If Jackson can build 7 birdhouses in 5 hours, write an equation that will allow Jackson to calculate the time it will take him to build any given number of birdhouses, assuming he works at a constant rate.

a. Write an equation that you could use to find out how long it will take him to build any number of birdhouses.

b. How many birdhouses can Jackson build in 40 hours?

c. How long will it take Jackson to build 35 birdhouses? Use the equation from part (a) to solve the problem.

d. How long will it take to build 71 birdhouses? Use the equation from part (a) to solve the problem.

Example 2: Al's Produce Stand

Al's Produce Stand sells 6 ears of corn for $\$1.50$. Barbara's Produce Stand sells 13 ears of corn for $\$3.12$. Write two equations, one for each produce stand, that models the relationship between the number of ears of corn sold and the cost. Then use each equation to help complete the tables below.

Al's Produce Stand

Ears	6	14	21	
Cost	$1.50			$50.00

Barbara's Produce Stand

Ears	13	14	21	
Cost	$3.12			$49.92

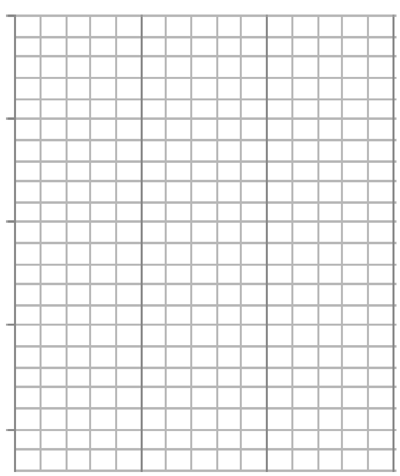
Lesson 10: Interpreting Graphs of Proportional Relationships

Classwork

Example 1

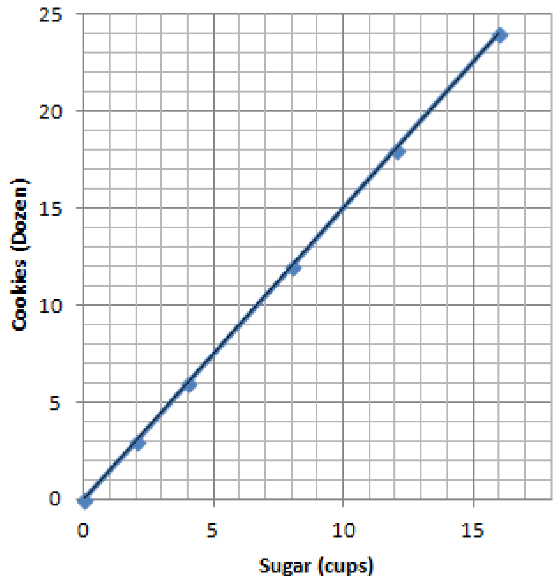
Grandma's Special Chocolate Chip Cookie recipe, which yields 4 dozen cookies, calls for 3 cups of flour.

Using this information, complete the chart:

Create a table comparing the amount of flour used to the amount of cookies.	Is the number of cookies proportional to the amount of flour used? Explain why or why not.	What is the unit rate of cookies to flour ($\frac{y}{x}$) and what is the meaning in the context of the problem?
Model the relationship on a graph. 	Does the graph show the two quantities being proportional to each other? Explain	Write an equation that can be used to represent the relationship.

Example 2

Below is a graph modeling the amount of sugar required to make Grandma's Chocolate Chip Cookies.



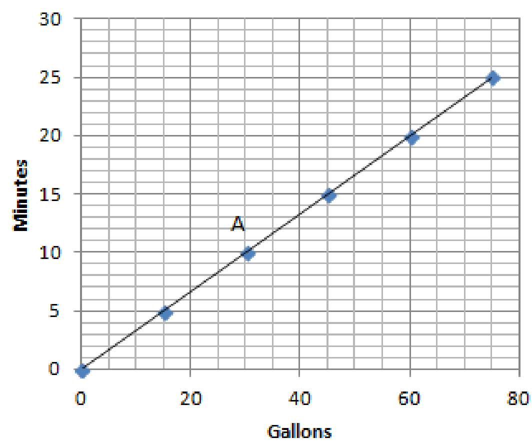
a. Record the coordinates from the graph in a table. What do these ordered pairs represent?

b. Grandma has 1 remaining cup of sugar. How many dozen cookies will she be able to make? Plot the point on the graph above.

c. How many dozen cookies can grandma make if she has no sugar? Can you graph this on the coordinate plane provided above? What do we call this point?

Exercises

1. The graph below shows the amount of time a person can shower with a certain amount of water.



   a. Can you determine by looking at the graph whether the length of the shower is proportional to the number of gallons of water? Explain how you know.

   b. How long can a person shower with 15 gallons of water? How long can a person shower with 60 gallons of water?

   c. What are the coordinates of point A? Describe point A in the context of the problem.

   d. Can you use the graph to identify the unit rate?

e. Plot the unit rate on the graph. Is the point on the line of this relationship?

f. Write the equation to represent the relationship between the number of gallons of water used and the length of a shower.

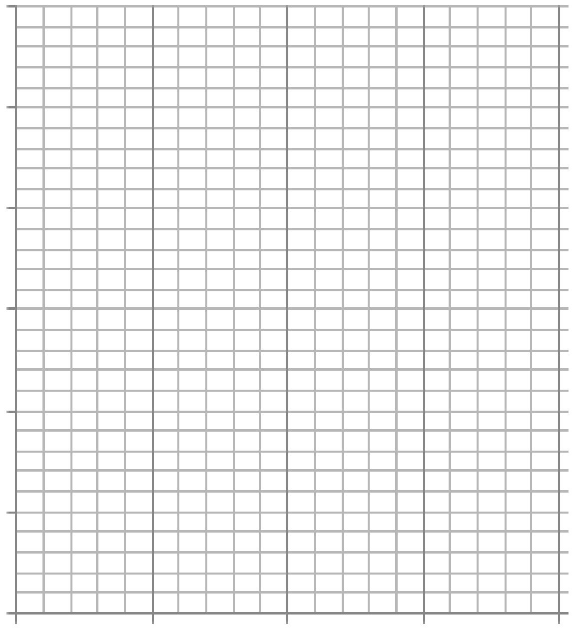
8. Your friend uses the equation $C = 50P$ to find the total cost, C, for the number of people, P, entering a local amusement park.

   a. Create a table and record the cost of entering the amusement park for several different-sized groups of people.

   b. Is the cost of admission proportional to the amount of people entering the amusement park? Explain why or why not.

   c. What is the unit rate and what does it represent in the context of the situation?

d. Sketch a graph to represent this relationship.



e. What points must be on the graph of the line if the two quantities represented are proportional to each other? Explain why and describe these points in the context of the problem.

f. Would the point $(5,250)$ be on the graph? What does this point represent in the context of the situation?

Lesson 11: Ratios of Fractions and Their Unit Rates

Classwork

Example 1: Who is Faster?

During their last workout, Izzy ran $2\frac{1}{4}$ miles in 15 minutes and her friend Julia ran $3\frac{3}{4}$ miles in 25 minutes. Each girl thought she was the faster runner. Based on their last run, which girl is correct? Use any approach to find the solution.

Example 2: Is Meredith Correct?

A turtle walks $\frac{7}{8}$ of a mile in 50 minutes. What is the unit rate expressed in miles per hour?

a. To find the turtle's unit rate, Meredith wrote the following complex fraction. Explain how the fraction $\frac{5}{6}$ was obtained.

$$\frac{\left(\frac{7}{8}\right)}{\left(\frac{5}{6}\right)}$$

b. Determine the unit rate, expressed in miles per hour.

Exercises

1. For Anthony's birthday, his mother is making cupcakes for his 12 friends at his daycare. The recipe calls for $3\frac{1}{3}$ cups of flour. This recipe makes $2\frac{1}{2}$ dozen cupcakes. Anthony's mother has only 1 cup of flour. Is there enough flour for each of his friends to get a cupcake? Explain and show your work.

2. Sally is making a painting for which she is mixing red paint and blue paint. The table below shows the different mixtures being used.

Red Paint (Quarts)	Blue Paint (Quarts)
$1\frac{1}{2}$	$2\frac{1}{2}$
$2\frac{2}{5}$	4
$3\frac{3}{4}$	$6\frac{1}{4}$
4	$6\frac{2}{3}$
1.2	2
1.8	3

a. What is the unit rate for the values of the amount of blue paint to the amount of red paint?

b. Is the amount of blue paint proportional to the amount of red paint?

c. Describe, in words, what the unit rate means in the context of this problem.

Lesson 12: Ratios of Fractions and Their Unit Rates

Classwork

During this lesson, you are remodeling a room at your house and need to figure out if you have enough money. You will work individually and with a partner to make a plan of what is needed to solve the problem. After your plan is complete, then you will solve the problem by determining if you have enough money.

Example 1: Time to Remodel

You have decided to remodel your bathroom and install a tile floor. The bathroom is in the shape of a rectangle and the floor measures 14 feet, 8 inches long by 5 feet, 6 inches wide. The tiles you want to use cost $\$5$ each, and each tile covers $4\frac{2}{3}$ square feet. If you have $\$100$ to spend, do you have enough money to complete the project?

Make a Plan: Complete the chart to identify the necessary steps in the plan and find a solution.

What I Know	What I Want to Find	How to Find it

Compare your plan with a partner. Using your plans, work together to determine how much money you will need to complete the project and if you have enough money.

Exercises

Which car can travel further on 1 gallon of gas?

Blue Car: travels $18\frac{2}{5}$ miles using 0.8 gallons of gas

Red Car: travels $17\frac{2}{5}$ miles using 0.75 gallons of gas

Lesson 13: Finding Equivalent Ratios Given the Total Quantity

Classwork

Example 1

A group of 6 hikers are preparing for a one-week trip. All of the group's supplies will be carried by the hikers in backpacks. The leader decides that each hiker will carry a backpack that is the same fraction of weight to all the other hikers' weights. This means that the heaviest hiker would carry the heaviest load. The table below shows the weight of each hiker and the weight of the backpack.

Complete the table. Find the missing amounts of weight by applying the same value of the ratio as the first two rows.

Hiker's Weight	Backpack Weight	Total Weight (lb.)
152 lb. 4 oz.	14 lb. 8 oz.	
107 lb. 10 oz.	10 lb. 4 oz.	
129 lb. 15 oz.		
68 lb. 4 oz.		
	8 lb. 12 oz.	
	10 lb.	

Example 2

When a business buys a fast food franchise, it is buying the recipes used at every restaurant with the same name. For example, all Pizzeria Specialty House Restaurants have different owners, but they must all use the same recipes for their pizza, sauce, bread, etc. You are now working at your local Pizzeria Specialty House Restaurant, and listed below are the amounts of meat used on one meat-lovers pizza.

$\frac{1}{4}$ cup of sausage

$\frac{1}{3}$ cup of pepperoni

$\frac{1}{6}$ cup of bacon

$\frac{1}{8}$ cup of ham

$\frac{1}{8}$ cup of beef

What is the total amount of toppings used on a meat-lovers pizza? _________________________ cups

The meat must be mixed using this ratio to ensure that customers will receive the same great tasting meat-lovers pizza from every Pizzeria Specialty House Restaurant nationwide. The table below shows 3 different orders for meat-lovers pizzas on Super Bowl Sunday. Using the amounts and total for one pizza given above, fill in every row and column of the table so the mixture tastes the same.

	Order 1	Order 2	Order 3
Sausage (cups)	1		
Pepperoni (cups)			3
Bacon (cups)		1	
Ham (cups)	$\frac{1}{2}$		
Beef (cups)			$1\frac{1}{8}$
TOTAL (cups)			

Exercise

The table below shows 6 different-sized pans that could be used to make macaroni and cheese. If the ratio of ingredients stays the same, how might the recipe be altered to account for the different sized pans?

Noodles (cups)	Cheese (cups)	Pan Size (number of cups)
		5
3	$\frac{3}{4}$	
	$\frac{1}{4}$	
$\frac{2}{3}$		
$5\frac{1}{3}$		
		$5\frac{5}{8}$

Lesson 14: Multi-Step Ratio Problems

Classwork

Example 1: Bargains

Peter's Pants Palace advertises the following sale: Shirts are $\frac{1}{2}$ off the original price; pants are $\frac{1}{3}$ off the original price, and shoes are $\frac{1}{4}$ off the original price.

a. If a pair of shoes cost $\$40$, what is the sales price?

b. At Peter's Pants Palace, a pair of pants usually sells for $\$33.00$. What is the sale price of Peter's pants?

Example 2: Big Al's Used Cars

A used car salesperson receives a commission of $\frac{1}{12}$ of the sales price of the car for each car he sells. What would the sales commission be on a car that sold for $\$21{,}999$?

Example 3: Tax Time

As part of a marketing plan, some businesses mark up their prices before they advertise a sales event. Some companies use this practice as a way to entice customers into the store without sacrificing their profits.

A furniture store wants to host a sales event to improve its profit margin and to reduce its tax liability before its inventory is taxed at the end of the year.

How much profit will the business make on the sale of a couch that is marked-up by $\frac{1}{3}$ and then sold at a $\frac{1}{5}$ off discount if the original price is $\$2{,}400$?

Example 4: Born to Ride

A motorcycle dealer paid a certain price for a motorcycle and marked it up by $\frac{1}{5}$ of the price he paid. Later he sold it for $\$14{,}000$. What is the original price?

Lesson 15: Equations of Graphs of Proportional Relationships Involving Fractions

Classwork

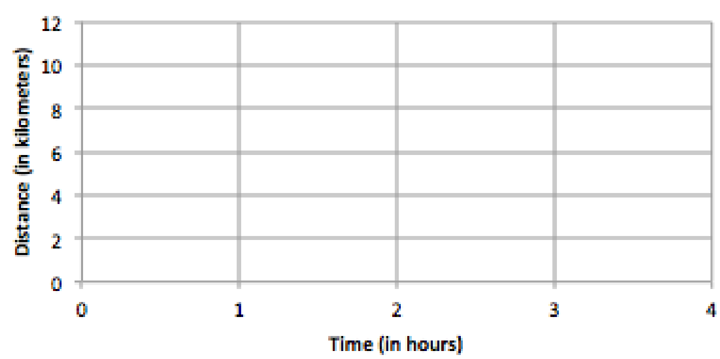
Example 1: Mother's 10K Race

Sam's mother has entered a 10K race. Sam and his family want to show their support of their mother, but they need to figure out where they should go along the race course. They also need to determine how long it will take her to run the race so that they will know when to meet her at the finish line. Previously, his mother ran a 5K race with a time of $1\frac{1}{2}$ hours. Assume Sam's mother ran the same rate as the previous race in order to complete the chart.

Create a table that will show how far Sam's mother has run after each half hour from the start of the race, and graph it on the coordinate plane to the right.

Time (H, in hours)	Distance Run (D, in km)

Mother's 10K Race



a. What are some specific things you notice about this graph?

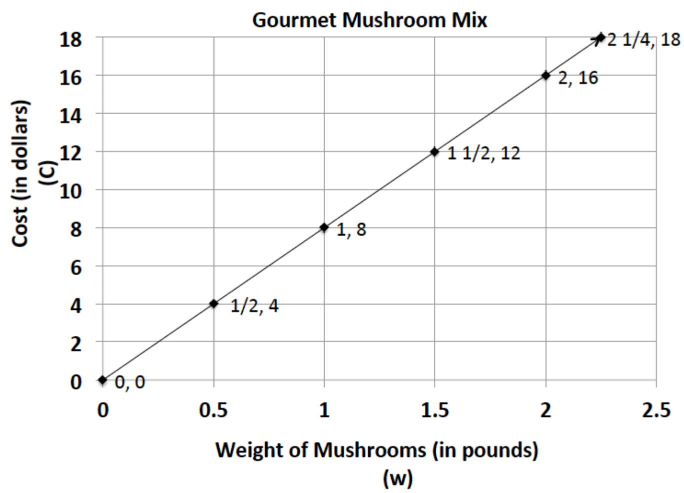
b. What is the connection between the table and the graph?

c. What does the ordered pair $\left(2, 6\frac{2}{3}\right)$ represent in the context of this problem?

Example 2: Gourmet Cooking

After taking a cooking class, you decide to try out your new cooking skills by preparing a meal for your family. You have chosen a recipe that uses gourmet mushrooms as the main ingredient. Using the graph below, complete the table of values and answer the following questions.

Weight (in pounds)	Cost (in dollars)
0	0
$\frac{1}{2}$	4
1	
$1\frac{1}{2}$	12
	16
$2\frac{1}{4}$	18



a. Is this relationship proportional? How do you know from examining the graph?

b. What is the unit rate for cost per pound?

c. Write an equation to model this data.

d. What ordered pair represents the unit rate, and what does it mean?

e. What does the ordered pair $(2, 16)$ mean in the context of this problem?

f. If you could spend $\$10.00$ on mushrooms, how many pounds could you buy?

g. What would be the cost of 30 pounds of mushrooms?

Lesson 16: Relating Scale Drawings to Ratios and Rates

Classwork

Opening Exercise: Can You Guess the Image?

1.

2.

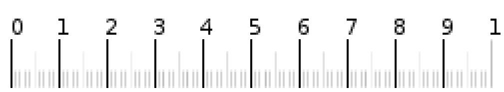
Example 1

For the following problems, (a) is the actual picture and (b) is the drawing. Is the drawing an enlargement or a reduction of the actual picture?

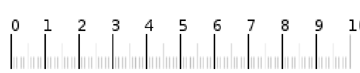
1. a.

b.

2. a.



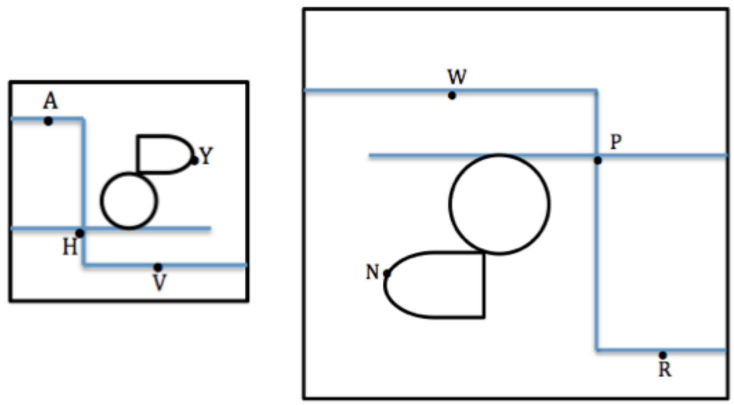
b.



Scale Drawing: a reduced or enlarged two-dimensional drawing of an original two-dimensional drawing.

Example 2

Derek's family took a day trip to a modern public garden. Derek looked at his map of the park that was a reduction of the map located at the garden entrance. The dots represent the placement of rare plants. The diagram below is the top-view as Derek held his map while looking at the posted map.

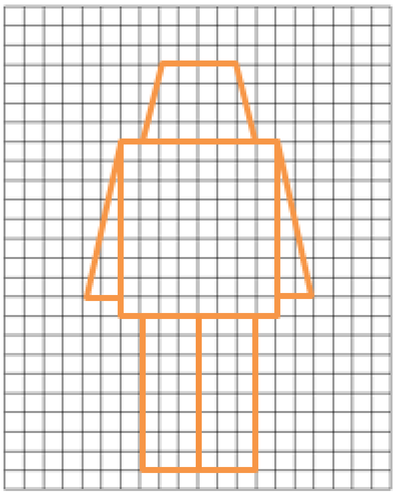


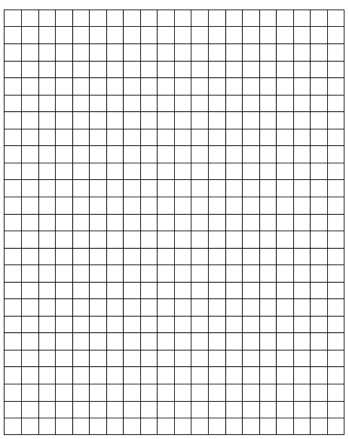
What are the corresponding points of the scale drawings of the maps?

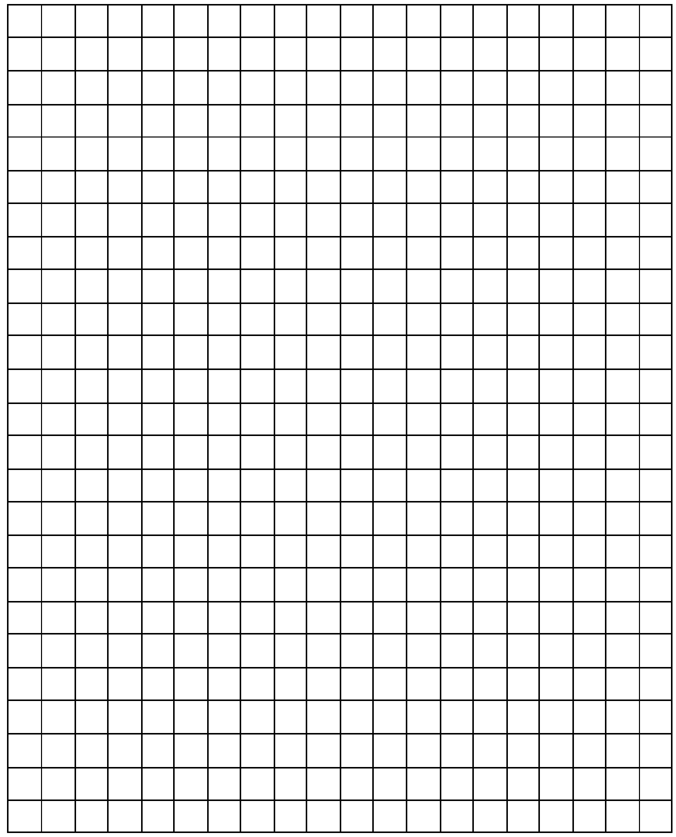
Point A to __________ Point V to _____________ Point H to _____________ Point Y to _____________

Exploratory Challenge

Create scale drawings of your own modern nesting robots using the grids provided.

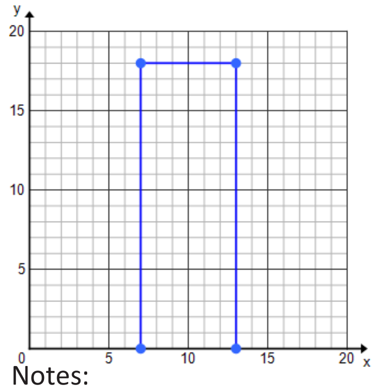


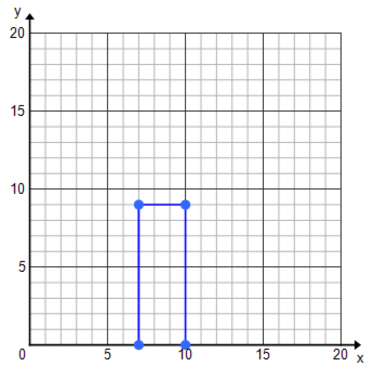




Example 3

Celeste drew an outline of a building for a diagram she was making and then drew a second one mimicking her original drawing. State the coordinates of the vertices and fill in the table.



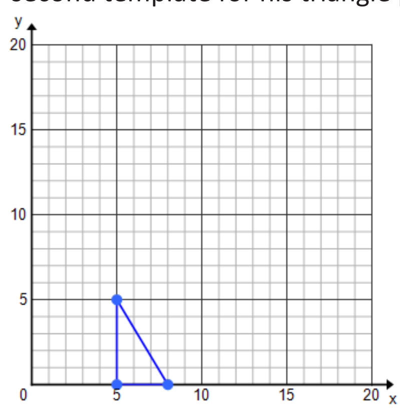


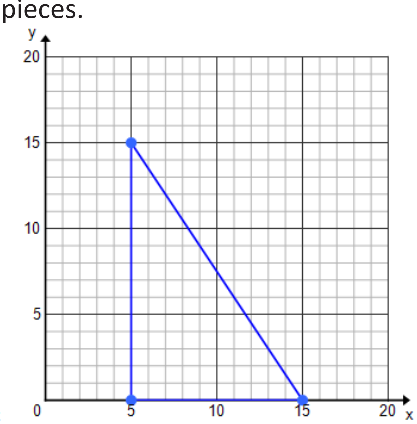
	Height	Length
Original Drawing		
Second Drawing		

Notes:

Exercise

Luca drew and cut out a small right triangle for a mosaic piece he was creating for art class. His mother really took a liking to the mosaic piece and asked if he could create a larger one for their living room. Luca made a second template for his triangle pieces.





	Height	Width
Original Image		
Second Image		

a. Does a constant of proportionality exist? If so, what is it? If not, explain.

b. Is Luca's enlarged mosaic a scale drawing of the first image? Explain why or why not.

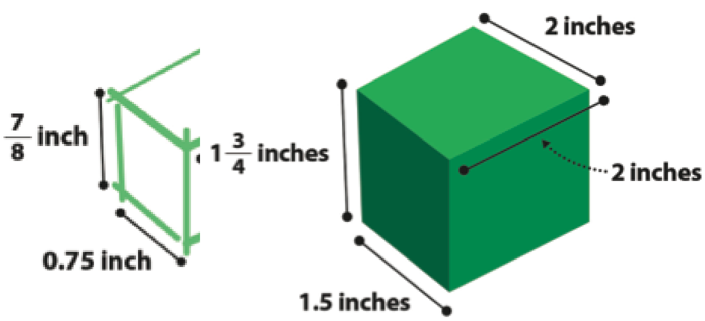
Lesson 17: The Unit Rate as the Scale Factor

Classwork

Example 1: Jake's Icon

Jake created a simple game on his computer and shared it with his friends to play. They were instantly hooked, and the popularity of his game spread so quickly that Jake wanted to create a distinctive icon so that players could easily identify his game. He drew a simple sketch. From the sketch, he created stickers to promote his game, but Jake wasn't quite sure if the stickers were proportional to his original sketch.

Original Sketch: **Sticker:**



Steps to check for proportionality for scale drawing and original object or picture:

1.

2.

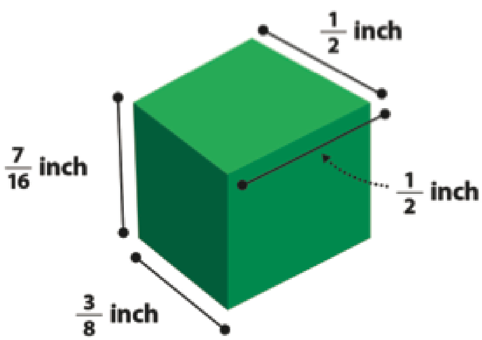
3.

Key Idea:

The **scale factor** can be calculated from the ratio of any length in the scale drawing to its corresponding length in the actual picture. The scale factor corresponds to the unit rate and the constant of proportionality.

Scaling by factors *greater than* 1 enlarge the segment, and scaling by factors *less than* 1, reduce the segment.

Exercise 1: App Icon



Example 2

Use a Scale Factor of 3 to create a scale drawing of the picture below.

Picture of the flag of Colombia:

Exercise 2

Scale Factor = ____________________________

Picture of the flag of Colombia:

Sketch and notes:

Example 3

Your family recently had a family portrait taken. Your aunt asks you to take a picture of the portrait using your phone and send it to her. If the original portrait is 3 feet by 3 feet, and the scale factor is $\frac{1}{18}$, draw the scale drawing that would be the size of the portrait on your phone.

Sketch and notes:

Exercise 3

John is building his daughter a doll house that is a miniature model of their house. The front of their house has a circular window with a diameter of 5 feet. If the scale factor for the model house is $\frac{1}{30}$, make a sketch of the circular doll house window.

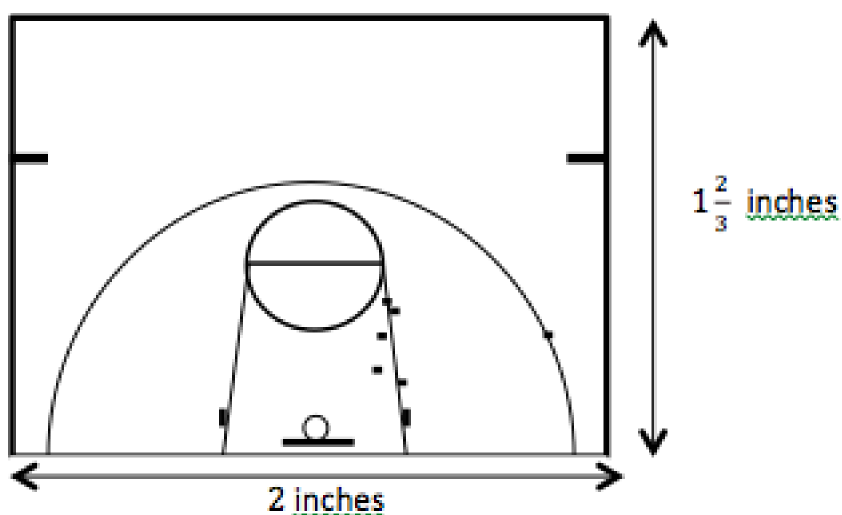
Lesson 18: Computing Actual Lengths from a Scale Drawing

Classwork

Example 1: Basketball at Recess?

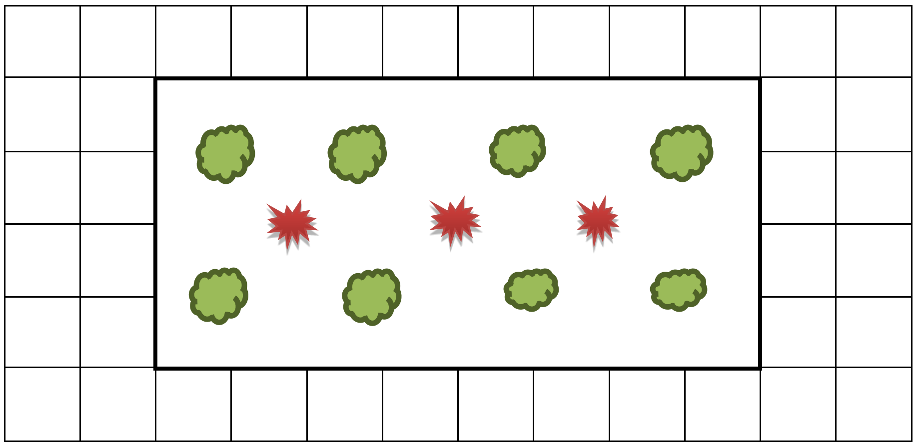
Vincent proposes an idea to the Student Government to install a basketball hoop along with a court marked with all the shooting lines and boundary lines at his school for students to use at recess. He presents a plan to install a half-court design as shown below. After checking with school administration, he is told it will be approved if it will fit on the empty lot that measures 25 feet by 75 feet on the school property. Will the lot be big enough for the court he planned? Explain.

Scale Drawing: 1 inch on the drawing corresponds to 15 feet of actual length.



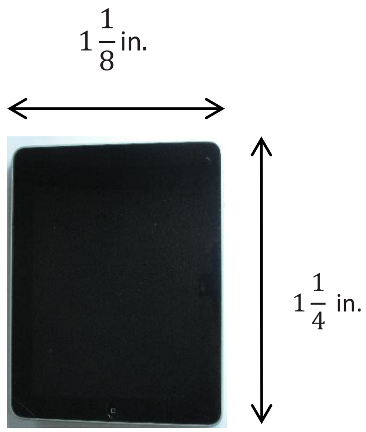
Example 2

The diagram shown represents a garden. The scale is 1 centimeter for every 20 meters. Each square in the drawing measures 1 cm by 1 cm. Find the actual length and width of the garden based upon the given drawing.



Example 3

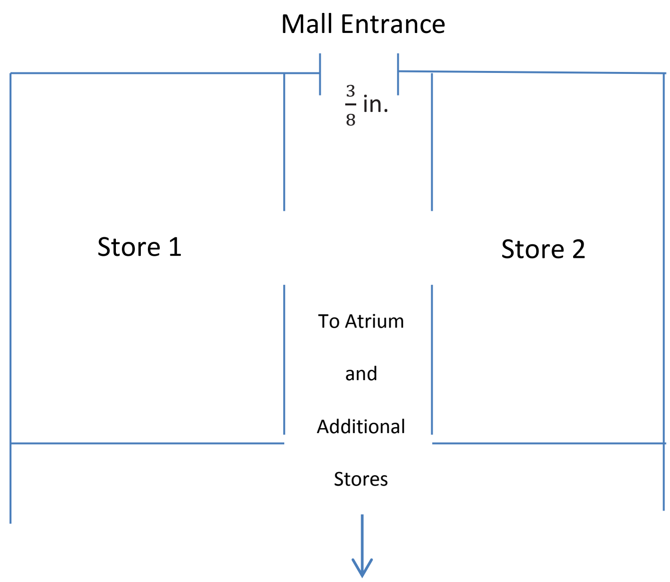
A graphic designer is creating an advertisement for a tablet. She needs to enlarge the picture given here so that 0.25 inches on the scale picture will correspond to 1 inch on the actual advertisement. What will be the length and width of the tablet on the advertisement?



Scale Picture of Tablet

Exercise

Students from the high school are going to perform one of the acts from their upcoming musical at the atrium in the mall. The students want to bring some of the set with them so that the audience can get a better feel for the whole production. The backdrop that they want to bring has panels that measure 10 feet by 10 feet. The students are not sure if they will be able to fit these panels through the entrance of the mall since the panels need to be transported flat (horizontal). They obtain a copy of the mall floor plan, shown below, from the city planning office. Use this diagram to decide if the panels will fit through the entrance. Use a ruler to measure.



Scale:

$\frac{1}{8}$ inch on the drawing represents $4\frac{1}{2}$ feet of actual length

Answer the following questions.

a. Find the actual distance of the mall entrance, and determine whether the set panels will fit.

b. What is the scale factor? What does it tell us?

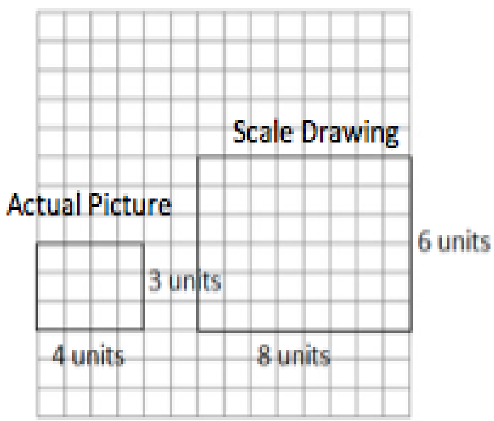
Lesson 19: Computing Actual Areas from a Scale Drawing

Classwork

Examples: Exploring Area Relationships

Use the diagrams below to find the scale factor and then find the area of each figure.

Example 1



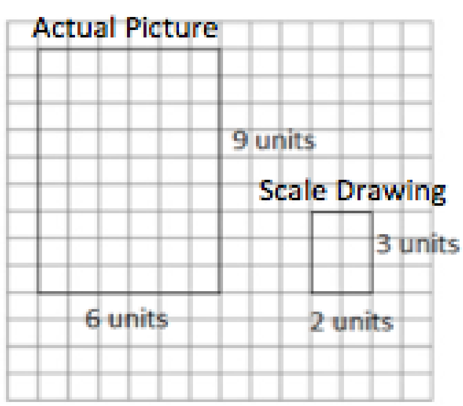
Scale factor: _________

Actual Area = ______________

Scale Drawing Area = ________________

Value of the Ratio of the Scale Drawing Area to the Actual Area: _________

Example 2



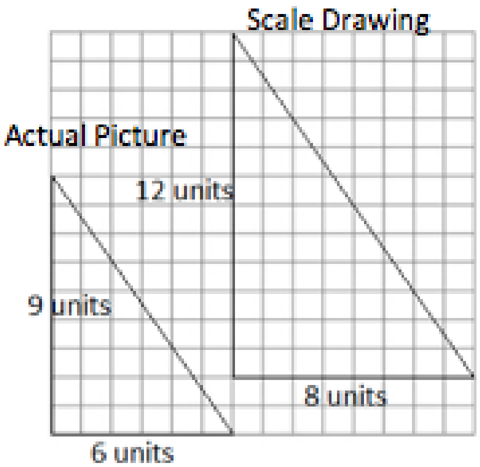
Scale factor: _________

Actual Area = ______________

Scale Drawing Area = ________________

Value of the Ratio of the Scale Drawing Area to the Actual Area: _________

Example 3



Scale factor: _________

Actual Area = ______________

Scale Drawing Area = ________________

Value of the Ratio of the Scale Drawing Area to the Actual Area: _________

Results: What do you notice about the ratio of the areas in Examples 1–3? Complete the statements below.

When the scale factor of the sides was 2, then the value of the ratio of the areas was _____________.

When the scale factor of the sides was $\frac{1}{3}$, then the value of the ratio of the areas was _____________.

When the scale factor of the sides was $\frac{4}{3}$, then the value of the ratio of the areas was _____________.

Based on these observations, what conclusion can you draw about scale factor and area?

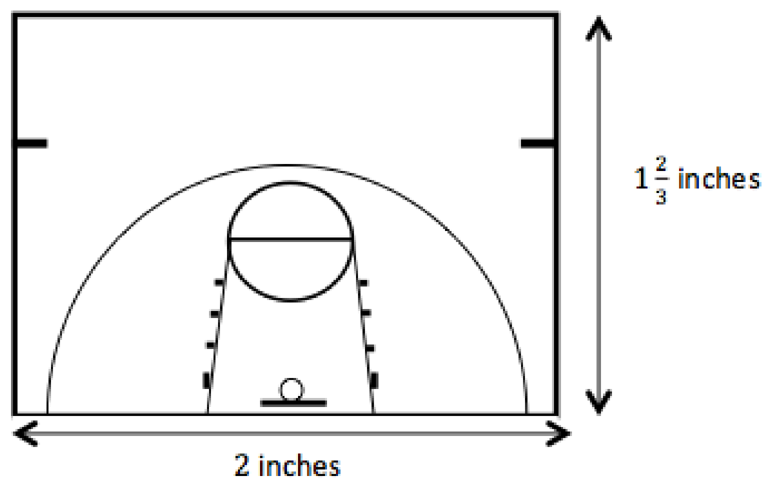
If the scale factor of the sides is r, then the ratio of the areas is ___________________.

Example 4: They Said Yes!

The Student Government liked your half-court basketball plan. They have asked you to calculate the actual area of the court so that they can estimate the cost of the project.

Based on your drawing below, what will the area of the planned half-court be?

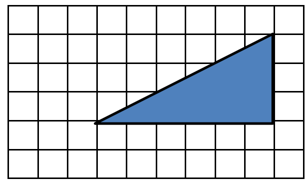
Scale Drawing: 1 inch on the drawing corresponds to 15 feet of actual length



Does the actual area you found reflect the results we found from Examples 1–3? Explain how you know.

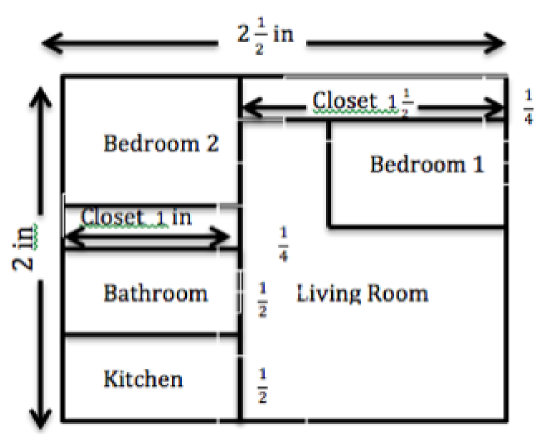
Exercises

1. The triangle depicted by the drawing has an actual area of 36 square units. What is the scale of the drawing? (Note: Each square on the grid has a length of 1 unit.)



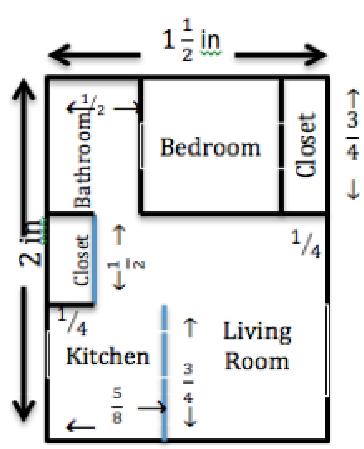
2. Use the scale drawings of two different apartments to answer the questions. Use a ruler to measure.

Suburban Apartment



Scale: 1 inch on scale drawing corresponds to 12 feet in the actual apartment.

City Apartment



Scale: 1 inch on scale drawing corresponds to 16 feet in actual apartment.

a. Find the scale drawing area for both apartments, and then use it to find the actual area of both apartments.

b. Which apartment has closets with more square footage? Justify your thinking.

c. Which apartment has the largest bathroom? Justify your thinking.

d. A one-year lease for the suburban apartment costs $\$750$ per month. A one-year lease for the city apartment costs $\$925$. Which apartment offers the greater value in terms of the cost per square foot?

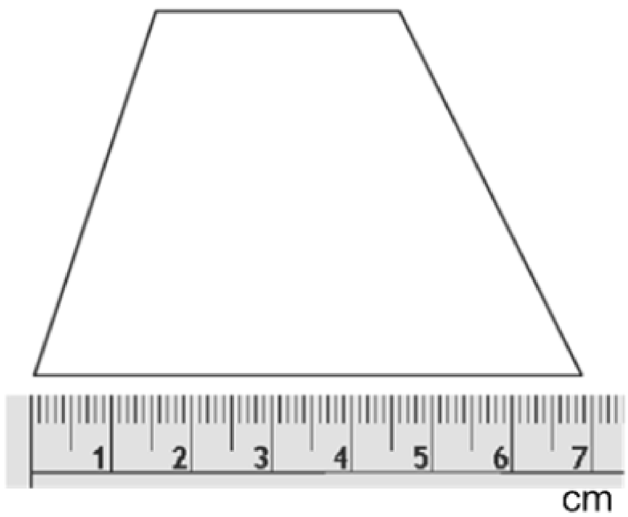
Lesson 21: An Exercise in Changing Scales

Classwork

How does your scale drawing change when a new scale factor is presented?

Exercise

The picture shows an enlargement or reduction of a scale drawing of a trapezoid.



Using the scale factor written on the card you chose, draw your new scale drawing with correctly calculated measurements.

a. What is the scale factor between the original scale drawing and the one you drew?

b. The longest base length of the actual trapezoid is 10 cm. What is the scale factor between original scale drawing and the actual trapezoid?

c. What is the scale factor between the new scale drawing you drew and the actual trapezoid?

Lesson 22: An Exercise in Changing Scales

Key Idea:

Two different scale drawings of the same top-view of a room are also scale drawings of each other. In other words, a scale drawing of a different scale can also be considered a scale drawing of the original scale drawing.

Example 1: Building a Bench

To surprise her mother, Taylor helped her father build a bench for the front porch. Taylor's father had the instructions with drawings but Taylor wanted to have her own copy. She enlarged her copy to make it easier to read. Using the following diagram, fill in the missing information. To complete the first row of the table, write the scale factor of the bench to the bench, the bench to the original diagram, and the bench to Taylor's diagram. Complete the remaining rows similarly.

The pictures below show the diagram of the bench shown on the original instructions and the diagram of the bench shown on Taylor's enlarged copy of the instruction.

Original Drawing of Bench (top view) Taylor's Drawing (top view) Scale factor to bench: $\frac{1}{12}$

2 inches

6 inches

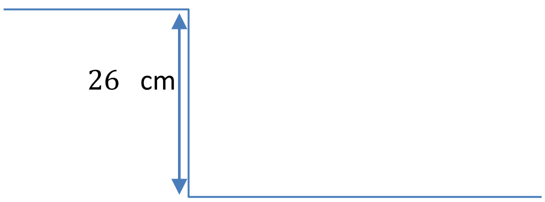
Scale Factors

	Bench	Original Diagram	Taylor's Diagram
Bench	1		
Original Diagram		1	
Taylor's Diagram			1

Exercise 1

Carmen and Jackie were driving separately to a concert. Jackie printed a map of the directions on a piece of paper before the drive, and Carmen took a picture of Jackie's map on her phone. Carmen's map had a scale factor to the actual distance of $\frac{1}{563{,}270}$. Using the pictures, what is the scale of Carmen's map to Jackie's map? What was the scale factor of Jackie's printed map to the actual distance?

Jackie's Map



Carmen's Map

Exercise 2

Ronald received a special toy train set for his birthday. In the picture of the train on the package, the boxcar has the following dimensions: length is $4\frac{5}{16}$ inches; width is $1\frac{1}{8}$ inches; and height is $1\frac{5}{8}$ inches. The toy boxcar that Ronald received has dimensions l is 17.25 inches; w is 4.5 inches; and h is 6.5 inches. If the actual boxcar is 50 feet long:

a. Find the scale factor of the picture on the package to the toy set.

b. Find the scale factor of the picture on the package to the actual boxcar.

c. Use these two scale factors to find the scale factor between the toy set and the actual boxcar.

d. What are the width and height of the actual boxcar?

Lesson 1: Percent

Classwork

Opening Exercise 1: Matching

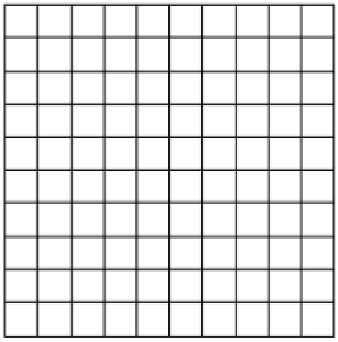
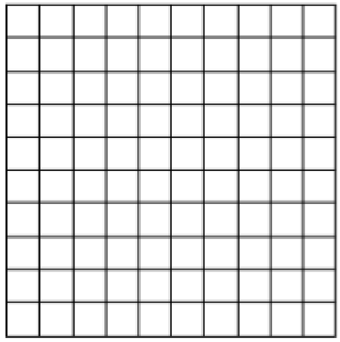
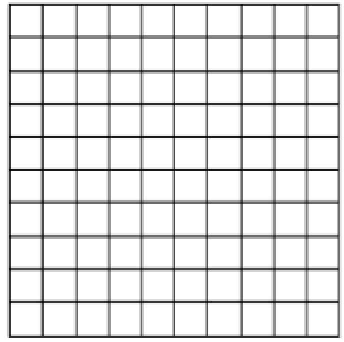
Match the percents with the correct sentence clues.

25%	I am half of a half. 5 cubic inches of water filled in a 20 cubic inch bottle.
50%	I am less than $\frac{1}{100}$. 25 out of $5{,}000$ contestants won a prize.
30%	I am the chance of birthing a boy or a girl. Flip a coin, and it will land on heads or tails.
1%	I am less than a half but more than one-fourth. 15 out of 50 play drums in a band.
10%	I am equal to 1. 35 question out of 35 questions were answered correctly.
100%	I am more than 1. Instead of the $\$1{,}200$ expected to be raised, $\$3{,}600$ was collected for the school's fundraiser.
300%	I am a tenth of a tenth. One penny is this part of one dollar.
$\frac{1}{2}\%$	I am less than a fourth but more than a hundredth. $\$11$ out of $\$110$ earned is saved in the bank.

Opening Exercise 2

Color in the grids to represent the following fractions:

a. $\frac{30}{100}$ b. $\frac{3}{100}$ c. $\frac{\frac{1}{3}}{100}$

Example 1

Use the definition of the word *percent* to write each percent as a fraction and then as a decimal.

Percent	Fraction	Decimal
37.5%		
100%		
110%		
1%		
$\frac{1}{2}\%$		

Example 2

Fill in the chart by converting between fractions, decimals, and percents. Show your work in the space below.

Fraction	Decimal	Percent
		350%
	0.025	
$\frac{1}{8}$		

Exercise: Class Card Activity

Read your card to yourself (each student will have a different card), and work out the problem. When the exercise begins, listen carefully to the questions being read. When you have the card with the equivalent value, respond by reading your card aloud.

Examples:

0.22 should be read "twenty two-hundredths."

$\frac{\frac{1}{5}}{1000}$ should be read "one-fifth thousandths" or "one-fifth over one thousand."

$\frac{7}{300}$ should be read "seven-three hundredths" or "seven over three hundred."

$\frac{200}{100}$ should be read "two hundred-hundredths" or "two hundred over one hundred."

Lesson 2: Part of a Whole as a Percent

Classwork

Opening Exercise

a. What is the whole unit in each scenario?

Scenario	Whole Unit
15 is what percent of 90?	
What number is 10% of 56?	
90% of a number is 180.	
A bag of candy contains 300 pieces and 25% of the pieces in the bag are red.	
Seventy percent (70%) of the students earned a B on the test.	
The 20 girls in the class represented 55% of the students in the class.	

b. Read each problem and complete the table to record what you know.

Problem	Part	Percent	Whole
40% of the students on the field trip love the museum. If there are 20 students on the field trip, how many love the museum?			
40% of the students on the field trip love the museum. If 20 students love the museum, how many are on the field trip?			
20 students on the field trip love the museum. If there are 40 students on the field trip, what percent love the museum?			

Example 1: Visual Approaches to Finding a Part, Given a Percent of the Whole

In Ty's math class, 20% of students earned an A on a test. If there were 30 students in the class, how many got an A?

Exercise 1

In Ty's art class, 12% of the Flag Day art projects received a perfect score. There were 25 art projects turned in by Ty's class. How many of the art projects earned a perfect score? (Identify the whole.)

Example 2: A Numeric Approach to Finding a Part, Given a Percent of the Whole

In Ty's English class, 70% of the students completed an essay by the due date. There are 30 students in Ty's English class. How many completed the essay by the due date?

Example 3: An Algebraic Approach to Finding a Part, Given a Percent of the Whole

A bag of candy contains 300 pieces of which 28% are red. How many pieces are red?

Which quantity represents the whole?

Which of the terms in the percent equation is unknown? Define a letter (variable) to represent the unknown quantity.

Write an expression using the percent and the whole to represent the number of pieces of red candy.

Write and solve an equation to find the unknown quantity.

Exercise 2

A bag of candy contains 300 pieces of which 28% are red. How many pieces are not red?

a. Write an equation to represent the number of pieces that are not red, n.

b. Use your equation to find the number of pieces of candy that are not red.

c. Jah-Lil told his math teacher that he could use the answer from part (b) and mental math to find the number of pieces of candy that are not red. Explain what Jah-Lil meant by that.

Example 4: Comparing Part of a Whole to the Whole with the Percent Formula

Zoey inflated 24 balloons for decorations at the middle school dance. If Zoey inflated 15% of the balloons that are inflated for the dance, how many balloons are there total? Solve the problem using the percent formula, and verify your answer using a visual model.

Example 5: Finding the Percent Given a Part of the Whole and the Whole

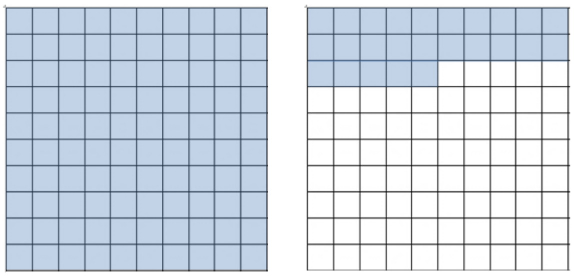
Haley is making admission tickets to the middle school dance. So far she has made 112 tickets, and her plan is to make 320 tickets. What percent of the admission tickets has Haley produced so far? Solve the problem using the percent formula, and verify your answer using a visual model.

Lesson 3: Comparing Quantities with Percent

Classwork

Opening Exercise

If each 10×10 unit square represents one whole, then what percent is represented by the shaded region?



In the model above, 25% represents a quantity of 10 students. How many students does the shaded region represent?

Example 1

a. The members of a club are making friendship bracelets to sell to raise money. Anna and Emily made 54 bracelets over the weekend. They need to produce 300 bracelets by the end of the week. What percent of the bracelets were they able to produce over the weekend?

b. Anna produced 32 bracelets of the 54 bracelets produced by Emily and Anna over the weekend. Compare the number of bracelets that Emily produced as a percent of those that Anna produced.

c. Write the number of bracelets that Anna produced as a percent of those that Emily produced.

Exercises 1–4

1. There are 750 students in the seventh-grade class and 625 students in the eighth-grade class at Kent Middle School.
   a. What percent is the seventh-grade class of the eighth-grade class at Kent Middle School?

   b. The principal will have to increase the number of eighth-grade teachers next year if the seventh-grade enrollment exceeds 110% of the current eighth-grade enrollment. Will she need to increase the number of teachers? Explain your reasoning.

2. At Kent Middle School, there are 104 students in the band and 80 students in the choir. What percent of the number of students in the choir is the number of students in the band?

3. At Kent Middle School, breakfast costs $\$1.25$ and lunch costs $\$3.75$. What percent of the cost of lunch is the cost of breakfast?

4. Describe a real-world situation that could be modeled using the equation $398.4 = 0.83(x)$. Describe how the elements of the equation correspond with the real-world quantities in your problem. Then, solve your problem.

Lesson 4: Percent Increase and Decrease

Classwork

Opening Exercise

Cassandra likes jewelry. She has five rings in her jewelry box.

a. In the box below, sketch Cassandra's five rings.

b. Draw a double number line diagram relating the number of rings as a percent of the whole set of rings.

c. What percent is represented by the whole collection of rings? What percent of the collection does each ring represent?

Example 1: Finding a Percent Increase

Cassandra's aunt said she will buy Cassandra another ring for her birthday. If Cassandra gets the ring for her birthday, what will be the percent increase in her ring collection?

Exercise 1

a. Jon increased his trading card collection by 5 cards. He originally had 15 cards. What is the percent increase? Use the equation $\text{Quantity} = \text{Percent} \times \text{Whole}$ to arrive at your answer, and then justify your answer using a numeric or visual model.

b. Suppose instead of increasing the collection by 5 cards, John increased his 15-card collection by just 1 card. Will the percent increase be the same as when Cassandra's ring collection increased by 1 ring (in Example 1)? Why or why not? Explain.

c. Based on your answer to part (b), how is displaying change as a percent useful?

Discussion

"I will only pay 90% of my bill."	"10% of my bill will be subtracted from the original total."

Example 2: Percent Decrease

Ken said that he is going to reduce the number of calories that he eats during the day. Ken's trainer asked him to start off small and reduce the number of calories by no more than 7%. Ken estimated and consumed $2,200$ calories per day instead of his normal $2,500$ calories per day until his next visit with the trainer. Did Ken reduce his calorie intake by 7%? Justify your answer.

Exercise 2

Skylar is answering the following math problem:

The value of an investment decreased by 10%. *The original amount of the investment was* $\$75.00$. *What is the current value of the investment?*

a. Skylar said 10% of $\$75.00$ is $\$7.50$, and since the investment decreased by that amount, you have to subtract $\$7.50$ from $\$75.00$ to arrive at the final answer of $\$67.50$. Create one algebraic equation that can be used to arrive at the final answer of $\$67.50$. Solve the equation to prove it results in an answer of $\$67.50$. Be prepared to explain your thought process to the class.

b. Skylar wanted to show the proportional relationship between the dollar value of the original investment, x, and its value after a 10% decrease, y. He creates the table of values shown. Does it model the relationship? Explain. Then, provide a correct equation for the relationship Skylar wants to model.

x	y
75	7.5
100	10
200	20
300	30
400	40

Example 3: Finding a Percent Increase or Decrease

Justin earned 8 badges in Scouts as of the Scout Master's last report. Justin wants to complete 2 more badges so that he will have a total of 10 badges earned before the Scout Master's next report.

a. If Justin completes the additional 2 badges, what will be the percent increase in badges?

b. Express the 10 badges as a percent of the 8 badges.

c. Does 100% plus your answer in part (a) equal your answer in part (b)? Why or why not?

Example 4: Finding the Original Amount Given a Percent Increase or Decrease

The population of cats in a rural neighborhood has declined in the past year by roughly 30%. Residents hypothesize that this is due to wild coyotes preying on the cats. The current cat population in the neighborhood is estimated to be 12. Approximately how many cats were there originally?

Example 5: Finding the Original Amount Given a Percent Increase or Decrease

Lu's math score on her achievement test in seventh grade was a 650. Her math teacher told her that her test level went up by 25% from her sixth grade test score level. What was Lu's test score level in sixth grade?

Closing

Phrase	Whole Unit (100%)
"Mary has 20% more money than John."	
"Anne has 15% less money than John."	
"What percent more (money) does Anne have than Bill?"	
"What percent less (money) does Bill have than Anne?"	

Lesson 5: Finding One Hundred Percent Given Another Percent

Classwork

Opening Exercise

What are the whole number factors of 100? What are the multiples of those factors? How many multiples are there of each factor (up to 100)?

Factors of 100	Multiples of the Factors of 100	Number of Multiples
100	100	1
50	$50,100$	2
1	$1, 2, 3, 4, 5, 6, \ldots, 98, 99, 100$	100

Example 1: Using a Modified Double Number Line with Percent

The 42 students who play wind instruments represent 75% of the students who are in band. How many students are in band?

Exercises 1–3

1. Bob's Tire Outlet sold a record number of tires last month. One salesman sold 165 tires, which was 60% of the tires sold in the month. What was the record number of tires sold?

2. Nick currently has $7{,}200$ points in his fantasy baseball league, which is 20% more points than Adam. How many points does Adam have?

3. Kurt has driven 276 miles of his road trip but has 70% of the trip left to go. How many more miles does Kurt have to drive to get to his destination?

Example 2: Mental Math Using Factors of 100

Answer each part below using only mental math, and describe your method.

a. If 39 is 1% of a number, what is that number? How did you find your answer?

b. If 39 is 10% of a number, what is that number? How did you find your answer?

c. If 39 is 5% of a number, what is that number? How did you find your answer?

d. If 39 is 15% of a number, what is that number? How did you find your answer?

e. If 39 is 25% of a number, what is that number? How did you find your answer?

Exercises 4–5

4. Derrick had a 0.250 batting average at the end of his last baseball season, which means that he got a hit 25% of the times he was up to bat. If Derrick had 47 hits last season, how many times did he bat?

5. Nelson used 35% of his savings account for his class trip in May. If he used $\$140$ from his savings account while on his class trip, how much money was in his savings account before the trip?

Lesson 6: Fluency with Percents

Classwork

Opening Exercise

Solve the following problem using mental math only. Be prepared to discuss your method with your classmates.

Cory and Everett have collected model cars since the third grade. Cory has 80 model cars in his collection, which is 25% more than Everett has. How many model cars does Everett have?

Example 1: Mental Math and Percents

a. 75% of the students in Jesse's class are 60 inches or taller. If there are 20 students in her class, how many students are 60 inches or taller?

b. Bobbie wants to leave a tip for her waitress equal to 15% of her bill. Bobbie's bill for her lunch is $\$18$. How much money represents 15% of the bill?

Exercises

1. Express 9 hours as a percentage of 3 days.

2. Richard works from 11:00 a.m. to 3:00 a.m. His dinner break is 75% of the way through his work shift. What time is Richard's dinner break?

3. At a playoff basketball game, there were 370 fans cheering for school A and 555 fans cheering for school B.
   a. Express the number of fans for school A as a percent of the number of fans for school B.

   b. Express the number of fans for school B as a percent of the number of fans for school A.

   c. What percent more fans were there for school B than for school A?

4. Rectangle A has a length of 8 cm and a width of 16 cm. Rectangle B has the same area as the first, but its length is 60% of the length of the first rectangle. Express the width of Rectangle B as a percent of the width of Rectangle A. What percent more or less is the width of Rectangle B than the width of Rectangle A?

5. A plant in Mikayla's garden was 40 inches tall one day and was 4 feet tall one week later. By what percent did the plant's height increase over one week?

6. Loren must obtain a minimum number of signatures on a petition before it can be submitted. She was able to obtain 672 signatures, which is 40% more than she needs. How many signatures does she need?

Lesson 7: Markup and Markdown Problems

Classwork

Example 1: A Video Game Markup

Games Galore Super Store buys the latest video game at a wholesale price of $\$30.00$. The markup rate at Game's Galore Super Store is 40%. You use your allowance to purchase the game at the store. How much will you pay, not including tax?

a. Write an equation to find the price of the game at Games Galore Super Store. Explain your equation.

b. Solve the equation from part (a).

c. What was the total markup of the video game? Explain.

d. You and a friend are discussing markup rate. He says that an easier way to find the total markup is by multiplying the wholesale price of $\$30.00$ by 40%. Do you agree with him? Why or why not?

Example 2: Black Friday

A $\$300$ mountain bike is discounted by 30%, and then discounted an additional 10% for shoppers who arrive before 5:00 a.m.

a. Find the sales price of the bicycle.

b. In all, by how much has the bicycle been discounted in dollars? Explain.

c. After both discounts were taken, what was the total percent discount?

d. Instead of purchasing the bike for $\$300$, how much would you save if you bought it before 5:00 a.m.?

Exercises 1–3

1. Sasha went shopping and decided to purchase a set of bracelets for 25% off of the regular price. If Sasha buys the bracelets today, she will receive an additional 5%. Find the sales price of the set of bracelets with both discounts. How much money will Sasha save if she buys the bracelets today?

2. A golf store purchases a set of clubs at a wholesale price of $\$250$. Mr. Edmond learned that the clubs were marked up 200%. Is it possible to have a percent increase greater than 100%? What is the retail price of the clubs?

3. Is a percent increase of a set of golf clubs from $\$250$ to $\$750$ the same as a markup rate of 200%? Explain.

Example 3: Working Backward

A car that normally sells for $\$20{,}000$ is on sale for $\$16{,}000$. The sales tax is 7.5%.

a. What percent of the original price of the car is the final price?

b. Find the discount rate.

c. By law, sales tax has to be applied to the discount price. However, would it be better for the consumer if the 7.5% sales tax was calculated before the 20% discount was applied? Why or why not?

d. Write an equation applying the commutative property to support your answer to part (c).

Exercise 4

a. Write an equation to determine the selling price in dollars, p, on an item that is originally priced s dollars after a markup of 25%.

b. Create and label a table showing five possible pairs of solutions to the equation.

c. Create and label a graph of the equation.

d. Interpret the points $(0,0)$ and $(1,r)$.

Exercise 5

Use the following table to calculate the markup or markdown rate. Show your work. Is the relationship between the original price and selling price proportional or not? Explain.

Original Price, m (in dollars)	Selling Price, p (in dollars)
$\$1{,}750$	$\$1{,}400$
$\$1{,}500$	$\$1{,}200$
$\$1{,}250$	$\$1{,}000$
$\$1{,}000$	$\$800$
$\$750$	$\$600$

Lesson 8: Percent Error Problems

Classwork

Example 1: How Far Off

Find the absolute error for the following problems. Explain what the absolute error means in context.

a. Taylor's Measurement 1

b. Connor's Measurement 1

c. Jordan's Measurement 2

Example 2: How Right Is Wrong?

a. Find the percent error for Taylor's Measurement 1. What does this mean?

b. From Example 1, part (b), find the percent error for Connor's Measurement 1. What does this mean?

c. From Example 1, part (c), find the percent error for Jordan's Measurement 2. What does it mean?

d. What is the purpose of finding percent error?

Exercises 1–3

Calculate the percent error for Problems 1–3. Leave your final answer in fraction form, if necessary.

1. A realtor expected 18 people to show up for an open house, but 25 attended.

2. In science class, Mrs. Moore's students were directed to weigh a 300-gram mass on the balance scale. Tina weighed the object and reported 328 grams.

3. Darwin's coach recorded that he had bowled 250 points out of 300 in a bowling tournament. However, the official scoreboard showed that Darwin actually bowled 225 points out of 300.

Example 3: Estimating Percent Error

The attendance at a musical event was counted several times. All counts were between 573 and 589. If the actual attendance number is between 573 and 589, inclusive, what is the most the percent error could be? Explain your answer.

Lesson 11: Tax, Commissions, Fees, and Other Real-World Percent Problems

Classwork

Opening Exercise

How are each of the following percent applications different, and how are they the same? Solve each problem, and then compare your solution process for each problem.

a. **Silvio earns 10% for each car sale he makes while working at a used car dealership. If he sells a used car for $\$2,000$, what is his commission?**

b. **Tu's family stayed at a hotel for 10 nights on their vacation. The hotel charged a 10% room tax, per night. How much did they pay in room taxes if the room cost $\$200$ per night?**

c. **Eric bought a new computer and printer online. He had to pay 10% in shipping fees. The items totaled $\$2,000$. How much did the shipping cost?**

d. **Selena had her wedding rehearsal dinner at a restaurant. The restaurant's policy is that gratuity is included in the bill for large parties. Her father said the food and service were exceptional, so he wanted to leave an extra 10% tip on the total amount of the bill. If the dinner bill totaled $\$2,000$, how much money did her father leave as the extra tip?**

Exercises

Show all work; a calculator may be used for calculations.

The school board has approved the addition of a new sports team at your school.

1. The district ordered 30 team uniforms and received a bill for $\$2,992.50$. The total included a 5% discount.
   a. The school needs to place another order for two more uniforms. The company said the discount will not apply because the discount only applies to orders of $\$1,000$ or more. How much will the two uniforms cost?

   b. The school district does not have to pay the 8% sales tax on the $\$2,992.50$ purchase. Estimate the amount of sales tax the district saved on the $\$2,992.50$ purchase. Explain how you arrived at your estimate.

   c. A student who loses a uniform must pay a fee equal to 75% of the school's cost of the uniform. For a uniform that cost the school $\$105$, will the student owe more or less than $\$75$ for the lost uniform? Explain how to use mental math to determine the answer.

   d. Write an equation to represent the proportional relationship between the school's cost of a uniform and the amount a student must pay for a lost uniform. Use u to represent the uniform cost and s to represent the amount a student must pay for a lost uniform. What is the constant of proportionality?

2. A taxpayer claims the new sports team caused his school taxes to increase by 2%.

   a. Write an equation to show the relationship between the school taxes before and after a 2% increase. Use b to represent the dollar amount of school tax before the 2% increase and t to represent the dollar amount of school tax after the 2% increase.

   b. Use your equation to complete the table below, listing at least 5 pairs of values.

b	t
$1{,}000$	
$2{,}000$	
	$3{,}060$
	$6{,}120$

   c. On graph paper, graph the relationship modeled by the equation in part (a). Be sure to label the axes and scale.

   d. Is the relationship proportional? Explain how you know.

   e. What is the constant of proportionality? What does it mean in the context of the situation?

   f. If a taxpayers' school taxes rose from $\$4{,}000$ to $\$4{,}020$, was there a 2% increase? Justify your answer using your graph, table, or equation.

3. The sports booster club sold candles as a fundraiser to support the new team. The club earns a commission on its candle sales (which means it receives a certain percentage of the total dollar amount sold). If the club gets to keep 30% of the money from the candle sales, what would the club's total sales have to be in order to make at least $\$500$?

4. Christian's mom works at the concession stand during sporting events. She told him they buy candy bars for $\$0.75$ each and mark them up 40% to sell at the concession stand. What is the amount of the markup? How much does the concession stand charge for each candy bar?

With your group, brainstorm solutions to the problems below. Prepare a poster that shows your solutions and math work. A calculator may be used for calculations.

5. For the next school year, the new soccer team will need to come up with $\$600$.

   a. Suppose the team earns $\$500$ from the fundraiser at the start of the current school year, and the money is placed for one calendar year in a savings account earning 0.5% simple interest annually. How much money will the team still need to raise to meet next year's expenses?

   b. Jeff is a member of the new sports team. His dad owns a bakery. To help raise money for the team, Jeff's dad agrees to provide the team with cookies to sell at the concession stand for next year's opening game. The team must pay back the bakery $\$0.25$ for each cookie it sells. The concession stand usually sells about 60 to 80 baked goods per game. Using your answer from part (a), determine a percent markup for the cookies the team plans to sell at next year's opening game. Justify your answer.

   c. Suppose the team ends up selling 78 cookies at next year's opening game. Find the percent error in the number of cookies that you estimated would be sold in your solution to part (b).

   Percent Error $= \frac{|a-x|}{|x|} \cdot 100\%$, where x is the exact value and a is the approximate value.

Lesson 16: Population Problems

Classwork

Opening Exercise

Number of girls in classroom:	Number of boys in classroom:	Total number of students in classroom:
Percent of the total number of students that are girls:	Percent of the total number of students that are boys:	Percent of boys and girls in the classroom:
Number of girls whose names start with a vowel:	Number of boys whose names start with a vowel:	Number of students whose names start with a vowel:
Percent of girls whose names start with a vowel:	Percent of boys whose names start with a vowel:	
Percent of the total number of students that are girls whose names start with a vowel:	Percent of the total number of students that are boys whose names start with a vowel:	Percent of students whose names start with a vowel:

Example 1

A school has 60% girls and 40% boys. If 20% of the girls wear glasses and 40% of the boys wear glasses, what percent of all students wears glasses?

Exercise 1

How does the percent of students who wear glasses change if the percent of girls and boys remains the same (that is, 60% girls and 40% boys), but 20% of the boys wear glasses and 40% of the girls wear glasses?

Exercise 2

How would the percent of students who wear glasses change if the percent of girls is 40% of the school and the percent of boys is 60% of the school, and 40% of the girls wear glasses and 20% of the boys wear glasses? Why?

Example 2

The weight of the first of three containers is 12% more than the second, and the third container is 20% lighter than the second. By what percent is the first container heavier than the third container?

Exercise 3

Matthew's pet dog is 7% heavier than Harrison's pet dog, and Janice's pet dog is 20% lighter than Harrison's. By what percent is Matthew's dog heavier than Janice's?

Example 3

In one year's time, 20% of Ms. McElroy's investments increased by 5%, 30% of her investments decreased by 5%, and 50% of her investments increased by 3%. By what percent did the total of her investments increase?

Exercise 4

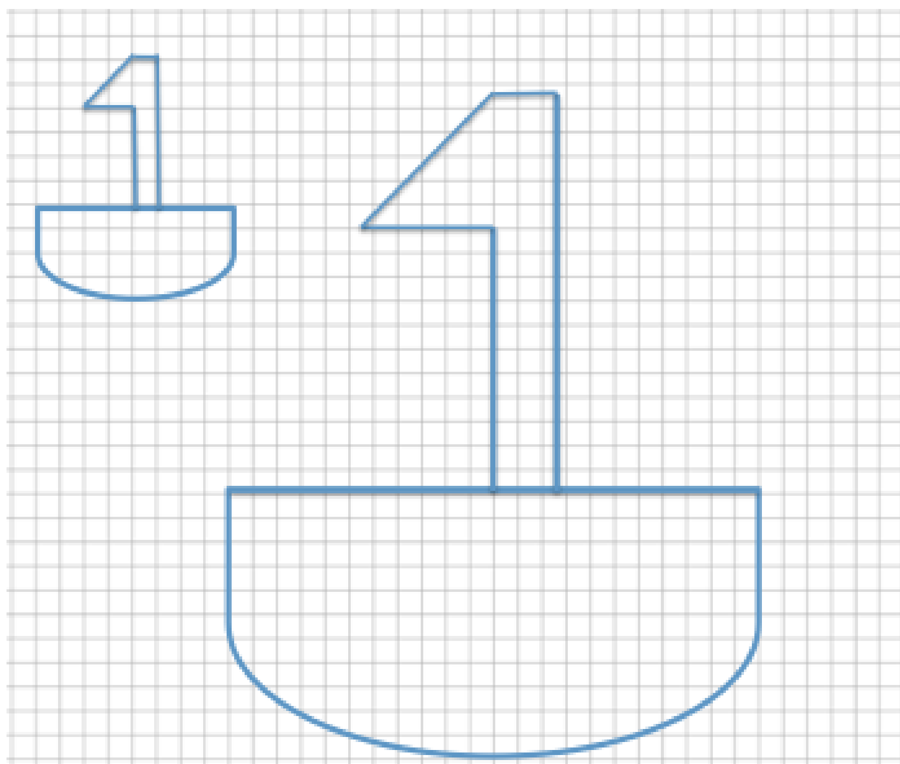
A concert had $6{,}000$ audience members in attendance on the first night and the same on the second night. On the first night, the concert exceeded expected attendance by 20%, while the second night was below the expected attendance by 20%. What was the difference in percent of concert attendees and expected attendees for both nights combined?

Lesson 14: Computing Actual Lengths from a Scale Drawing

Classwork

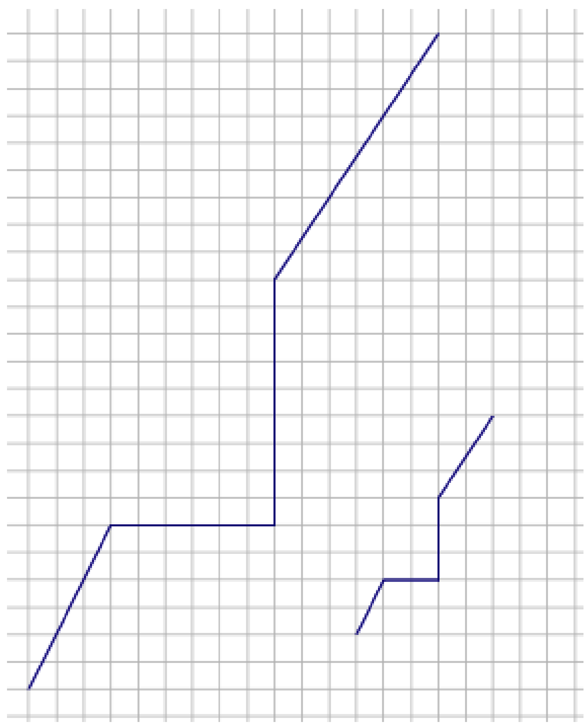
Example 1

The distance around the entire small boat is 28.4 units. The larger figure is a scale drawing of the smaller drawing of the boat. State the scale factor as a percent, and then use the scale factor to find the distance around the scale drawing.

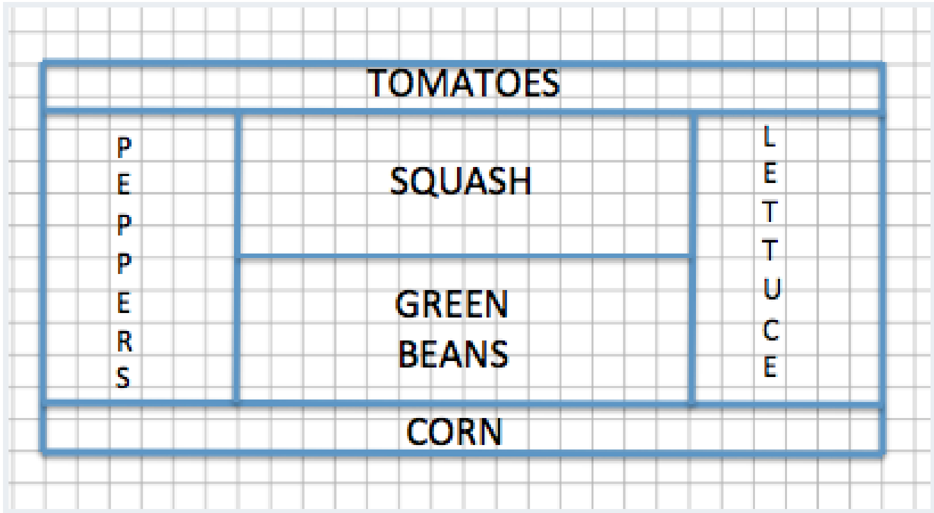


Exercise 1

The length of the longer path is 32.4 units. The shorter path is a scale drawing of the longer path. Find the length of the shorter path, and explain how you arrived at your answer.



Example 2: Time to Garden



Sherry designed her garden as shown in the diagram above. The distance between any two consecutive vertical grid lines is 1 foot, and the distance between any two consecutive horizontal grid lines is also 1 foot. Therefore, each grid square has an area of one square foot. After designing the garden, Sherry decided to actually build the garden 75% of the size represented in the diagram.

a. What are the outside dimensions shown in the blueprint?

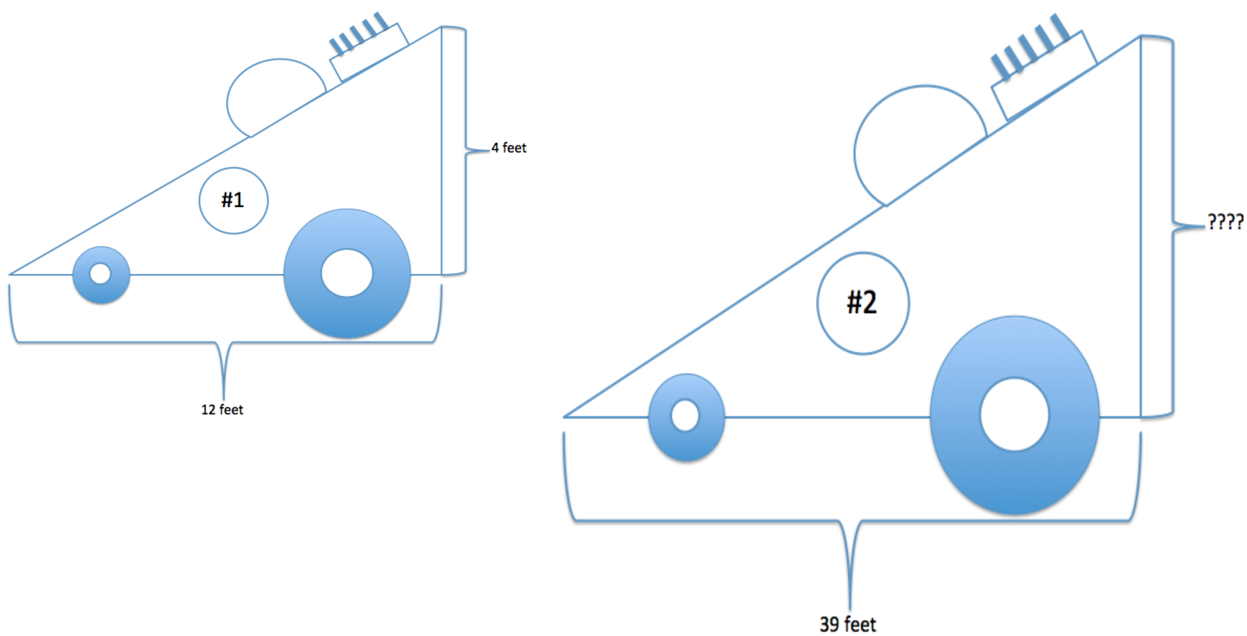
b. What will the overall dimensions be in the actual garden? Write an equation to find the dimensions. How does the problem relate to the scale factor?

c. If Sherry plans to use a wire fence to divide each section of the garden, how much fence does she need?

d. If the fence costs \$3.25 per foot plus 7% sales tax, how much would the fence cost in total?

Example 3

Race Car #2 is a scale drawing of Race Car #1. The measurement from the front of Car #1 to the back of Car #1 is 12 feet, while the measurement from the front of Car #2 to the back of Car #2 is 39 feet. If the height of Car #1 is 4 feet, find the scale factor, and write an equation to find the height of Car #2. Explain what each part of the equation represents in the situation.



Exercise 2

Determine the scale factor, and write an equation that relates the vertical heights of each drawing to the scale factor. Explain how the equation illustrates the relationship.

Exercise 3

The length of a rectangular picture is 8 inches, and the picture is to be reduced to be $45\frac{1}{2}\%$ of the original picture. Write an equation that relates the lengths of each picture. Explain how the equation illustrates the relationship.